Theoretical feasibility of using machine learning to determine experimental observables sensitive to the asymmetry energy

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Heavy-ion collisions provide an important probe of the nuclear equation-of-state (EoS). However, the dependence of different observables on the underlying interaction is not always clearly defined. A multidimensional analysis technique has been used to discriminate the observables sensitive to the asymmetry energy based on the results from the interaction of ^{124}Sn+^{64}Ni at 15 MeV/nucleon simulated from within Constrained Molecular Dynamics (CoMD) [1] and a Stochastic Mean Field (SMF) model [2, 3]. This multidimensional technique can be used to enhance our analysis of experimental observables and, thus, improve our ability to constrain the nuclear equation of state.

As experimental data sets in physics become larger and more complicated, there is an increasing need for data analysis methods that can treat the data efficiently and in an unbiased manner. This is most apparent as we refine our understanding, and focus our attention on finer details. In such cases, examining several observables simultaneously can lead to a consistent physical picture, where examining a single observable leaves room for uncertainty. The community has recognized this issue of the need to consider multiple variables simultaneously instead of treating them one-by-one. The approach is based on a modern version of a classical approach [4] to multi-variate statistical analysis, namely using the Sliced Inverse Regression algorithm [5, 6] within the R statistical environment[7]. The Sliced Inverse Regression Method (SIR) offers the ability to efficiently and, in an unbiased way, perform such analysis. We have demonstrated how the SIR method may be applied to a currently relevant topic in nuclear physics, namely constraining the asymmetry energy in the nuclear equation of state.

Approximately 3,000 events were simulated with SMF to train the SIR algorithm in identifying the way observables behave relative to a change in E_{sym}. The output of SMF (flat impact parameter distribution from 6 to 8 fm) was then treated with a coalescence code[8] to identify the free nucleons and clusters that appear in the exit channel based on the locations and proximity of the test particles relative to each other in phase space. The coalescence code output was then filtered in order to select only on the projectile-like fragment (PLF) at t=450 fm/c. The data was then filtered with a geometric software filter for the Forward Array Using Silicon Technology (FAUST)[9]. This was done to examine the ability of the SIR method to find observables that may be reasonable through experimental techniques for detection.

The observables selected for use in the SIR method were determined from those proposed by theory to be sensitive to the EoS [3, 10–15]. These observables include the mass (A), charge (Z), excitation energy ($E*$), spin (J), center of mass momentum vector components (p_x , p_y , p_z), position in the center of mass frame (x, y, z), distance from the center of the fragment to the center of mass frame ($r2$), and the quadrupole ($Quad_{mom}$) and octupole (Oct_{mom}) moments, all of the PLF. Fig. 1

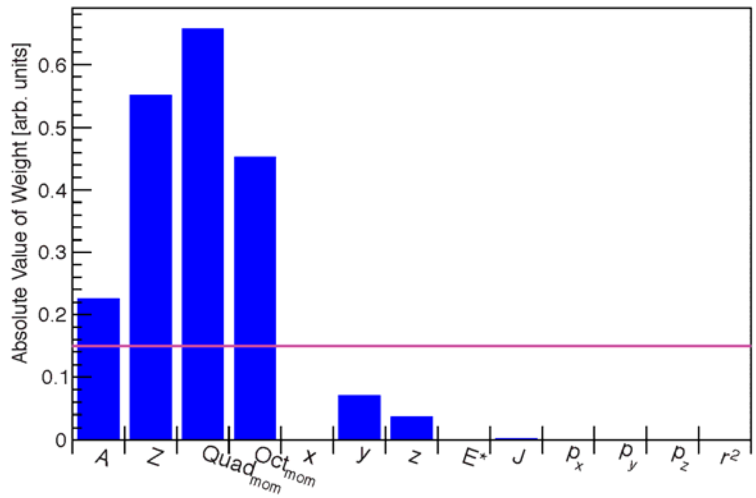

FIG. 1. Absolute values of the observable weights as determined by SIR for the case of 2 E*sym* at an impact parameter of 6 fm using SMF. The solid pink line, equivalent to an observable weight of 0.15, indicates the cut-off used to determine the most important observables. This value is arbitrarily defined and may vary depending on the analysis.

shows the absolute value of the weighted coefficients for each of the 13 observables. The most important observables were chosen to be those that had a weight greater than 0.15, represented by the solid red line. This value is arbitrarily defined and may vary depending on the analysis. In this regard, the mass, charge and deformation in momentum space of the PLF (quadrupole and octupole moments) were the most significant terms. By looking at the distributions of the observables for the PLFs individually, it is difficult to make a clear determination as to how they are affected by the asymmetry energy. In Fig. 2, both stiff and soft forms of E_{sym} are overlaid for each observable from SMF that was determined to be significant. A close inspection shows there is clearly no real separation in the mean value of each observable for the PLFs using the stiff and soft asymmetry energy from SMF.

Re-analysis of the same data this time using only the principal observables as determined previously by SIR (shown above the red line in Fig. 1 as A, Z, $Quad_{mom}$ and Oct_{mom}) provided a clear separation between the mean SIR_{value} for the stiff and soft forms of E_{sym} in SMF, as shown in Fig. 3. This emphasizes that analyzing multiple observables together yields a better understanding and more complete picture of the effect of the asymmetry energy on the observables. The analysis, using only the principal observables, yields the function

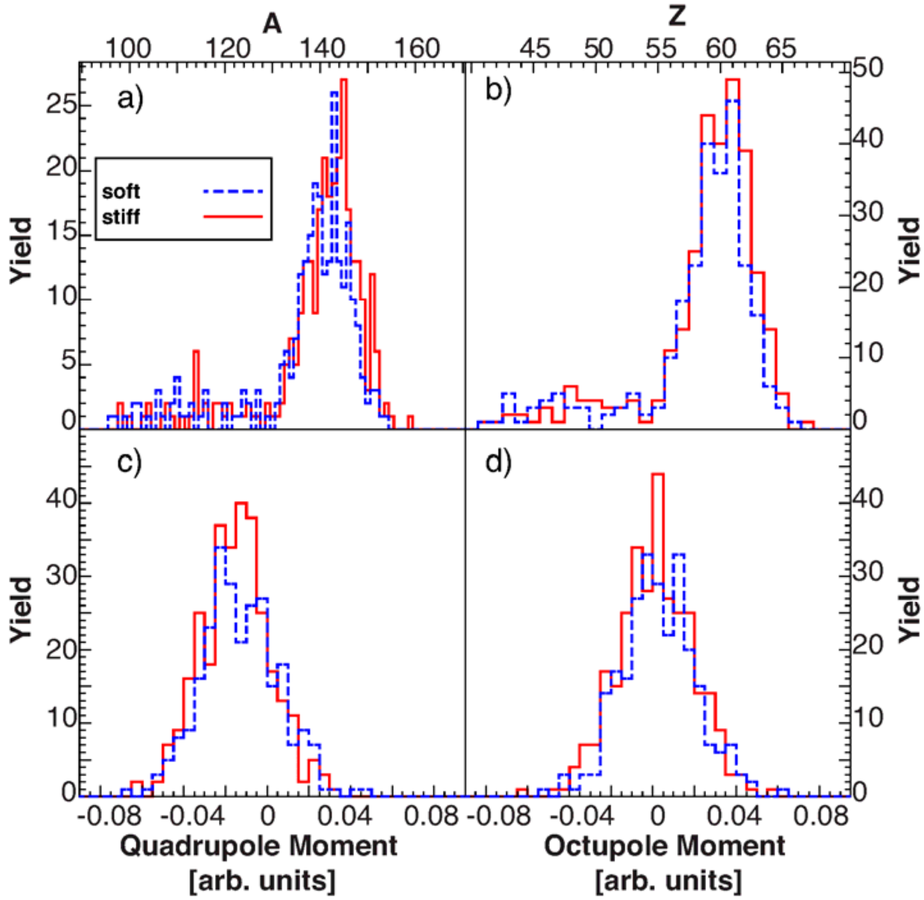

FIG. 2. Mass (a), Charge (b), (c) Quadrupole Moment and (d) Octupole Moment for stiff (solid red) and soft (dashed blue) E_{sym} at an impact parameter of 6fm. All data shown is for PLFs resulting from a 2-body (binary) breakup of the system at t=450 fm/c using the SMF model.

$$SIR_{value} = -0.7338A + 0.17392Z + 0.97921Quad_{mom} - 0.07429Oct_{mom} + constan \qquad (1)$$

where the SIR_{value} is a value based on the linear combination of the weighted observables in arbitrary units. The same methodology was applied to similar observables in CoMD with similar results [16].

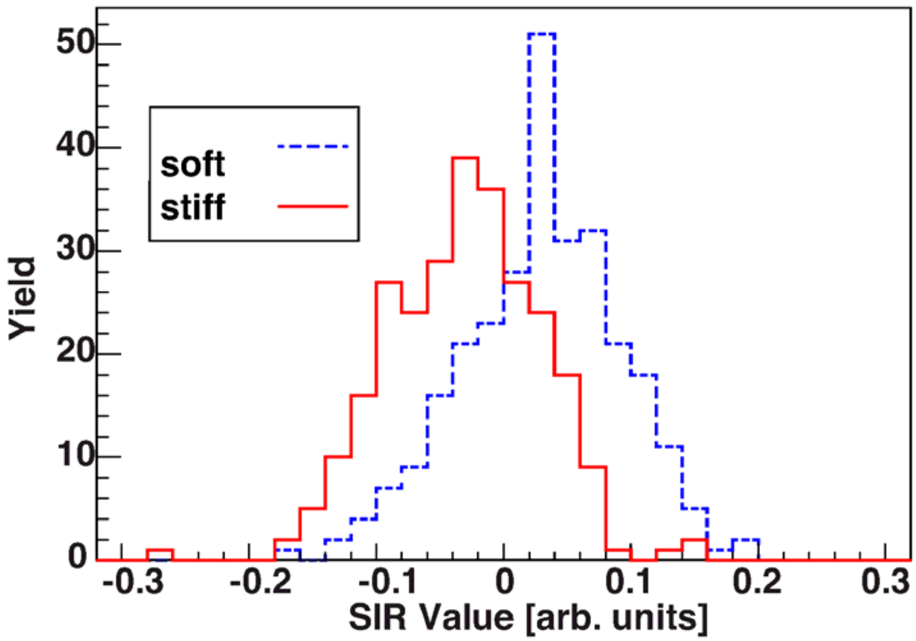

FIG. 3. Separation of the asymmetry energy via SIR for the projectile-like fragment from SMF. The dashed blue line represents the soft and the solid red line represents the stiff E_{sym} at an impact parameter of 6 fm.

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